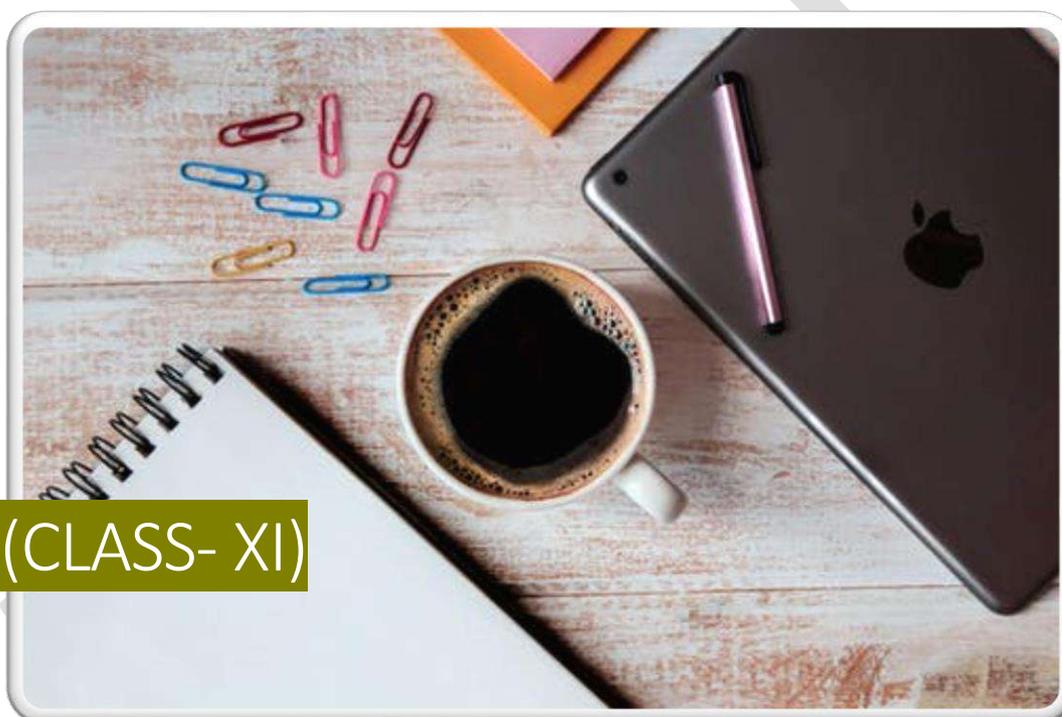




PHYSICS NOTES



(CLASS- XI)

(Class-XI) CHAPTER -3

MOTION IN A STRAIGHT LINE



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REST, MOTION AND REFERENCE FRAME

Kinematics is the branch of physics that deals with motion of bodies without inquiring its cause (the concept of forces).

A Particle is defined as matter of infinitesimally small size. Thus a particle has only a definite position but no dimension.

A certain amount of matter limited in all directions with finite size, shape occupying some definite space, is called a **Body**.



- **Rest**

*A body is said to be at **rest** when it doesn't change its position with time.*

Example- A blackboard sticking to a wall for some time is said to be at rest when it doesn't move from its position.

- **Motion**

*A body is said to be in **motion** if the position of the body continuously changes with time. Example- Movement of a car w.r.t. an observer at rest.*

- **Frame of Reference :**

*To locate the position of a body relative to the reference body a system of coordinates fixed on the reference body is constructed. This is known as **reference frame**.*

If two cars A and B move side by side in same direction with same speed, it would appear to the passengers of the cars that they are mutually at rest. Obviously, B is at rest relative to A. The reverse is also true.



Absolute rest or absolute motion is undefined. Motions are relative.

****NOTE:** Description of state of motion of a particle requires a set of axis, w.r.t to which the state must be specified, otherwise it won't make any sense. The inherent meaning of this statement is that we need a **reference frame** to determine whether a body is at rest or in motion.*

IMPORTANT TERMS AND DEFINITIONS

Terms	Definitions
Rectilinear Motion	A body is said to have a rectilinear motion when any two particles of a body travel the same distance along two straight parallel lines
Position vector	It describes the instantaneous position of a particle with respect to chosen frame of reference. It is a vector joining the origin of a particle. For one dimensional it's defined as : $r = x \hat{i}$ For two dimensional : $r = x \hat{i} + y \hat{j}$ For three dimensional : $r = x \hat{i} + y \hat{j} + z \hat{k}$
Distance	The total path length covered by the particle in a given time interval is known as distance .
Displacement	The vector joining the initial and final positions of the particle during a given time interval is called displacement . It is the change in position of a moving object.
Speed	The rate of change of distance with time is known as speed .
Velocity	The rate of change of displacement with respect to time is known as velocity .
Acceleration	The rate of change of velocity with respect to time is known as acceleration .
Uniform Motion	A body is said to be in uniform motion if it travels equal distance in equal intervals of time. It means the velocity remains constant during the motion.
Non-Uniform Motion	A body is said to be in non-uniform motion if it travels unequal distance in equal intervals of time.

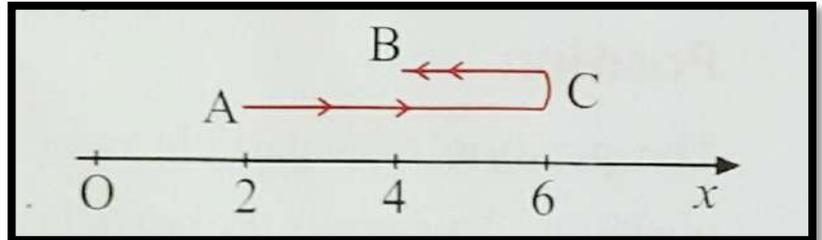
DISTANCE AND DISPLACEMENT

What is Distance?

Let a particle has a starting point at 'A=2m' and later it comes to final point 'B=4m' after turning round through 'C=6m'.

The distance travelled here =

$$AC + BC = 4 + 2 = 6\text{m}$$



"Distance is the total path length of the travelled by the particle in a given time interval."

- Path length of a body is a positive scalar quantity which doesn't decrease with time and can never be zero for any moving body.
- Magnitude of distance is greater than or equal to the magnitude of displacement.

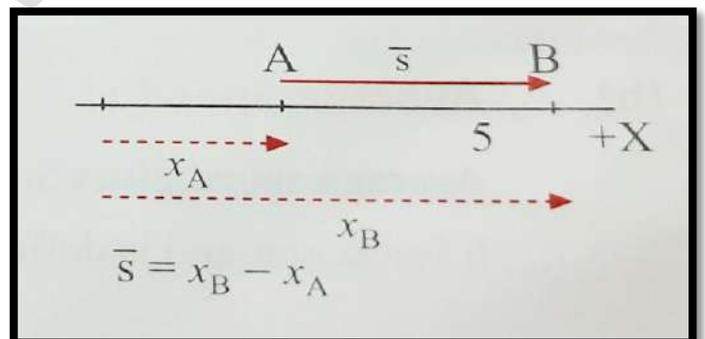
What is Displacement?

It is the vector joining the initial and final position of an object during a time interval. The change in position of moving object is known as displacement.

- For a straight line motion, if a particle goes from A to B then, $s = \text{displacement} = AB$, the vector has only x component.

Hence its equal to the difference in X coordinates

$$S = x_b - x_a = \Delta X$$



- If a particle goes from A to B along a curve in some time duration and if O is the origin then,

$$\overline{OA} = \text{initial position vector} = \vec{r}_i$$

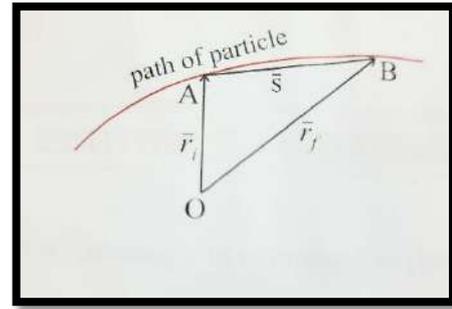
$$\overline{OB} = \text{final position vector} = \vec{r}_f$$

AB= displacement of vector = $\overline{OB} - \overline{OA}$

$$\vec{s} = \vec{r}_f - \vec{r}_i$$

$$|\vec{s}| = (|\vec{OB}^2| - |\vec{OA}^2|)^{1/2}$$

$$|\vec{s}| = \sqrt{OB^2 - OA^2}$$



Distance vs Displacement

Distance	Displacement
Distance is the total path length of the travelled by the particle in a given time interval.	It is the vector joining the initial and final position of an object during a time interval. The change in position of moving object is known as displacement.
It's a positive scalar quantity.	It's a vector quantity.
It has only magnitude.	It has both magnitude and direction.
It cannot be zero during motion. In case of circular motion if a body completes 2π rotation then the distance is $2\pi r$	It can be zero during motion. In case of a circular motion, if anybody completes 2π rotation the displacement is 0
It is always greater than or equal to the magnitude of displacement.	It's less than or equal to the magnitude of distance.

Example:

A body moves 6m north, 8m east and 10m vertically upwards. What is the magnitude of the resultant displacement from initial position?

Ans : Magnitude of the resultant displacement = $(AB^2 + BC^2)^{1/2} = 10 \cdot 2^{1/2} = 10\sqrt{2}$



AVERAGE AND INSTANTANEOUS VELOCITY/SPEED

- **AVERAGE SPEED:** It tells us how fast a particle moves in a particular interval.

It is a scalar quantity and is defined over an interval as Average Speed = Total Distance Covered/ Total Time Interval

Average speed has unit (m s^{-1}) and the magnitude of average speed is always greater than or equal to magnitude of average velocity.

- **AVERAGE VELOCITY:** The average velocity of a moving particle over a certain time interval is defined as the displacement divided by the time duration.

$$\text{Average Velocity} = \frac{\vec{\Delta r}}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j}$$

For straight line motion along X axis, we have

$$V_{\text{avg}} = \frac{\Delta x}{\Delta t} \hat{i}$$

The average velocity is the vector in the direction of displacement. It depends only on the net displacement and time interval and not on the journey.

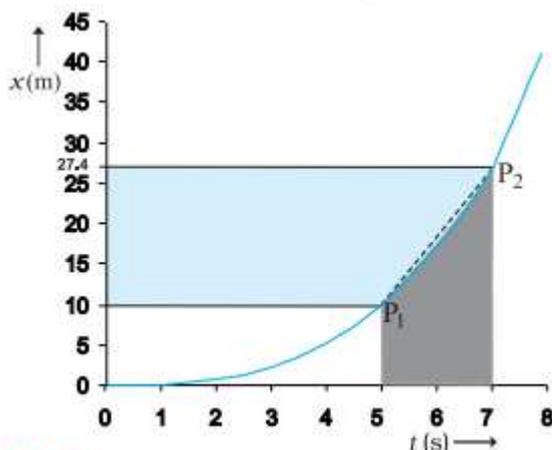


Fig. 3.4 The average velocity is the slope of line P_1P_2 .

$$V_{\text{avg}} = (27.4 - 10)/(7-5) \\ = 8.7 \text{ ms}^{-1}$$

Example:

A car is moving along a straight line, say OP in Fig. 3.1. It moves from O to P in 18 s and returns from P to Q in 6.0 s. What are the average velocity and average speed of the car in going (a) from O to P? And (b) from O to P and back to Q?

Ans : (a) Average Velocity = Displacement / Time Interval

$$\vec{v} = + 360 / 18 = 20 \text{ m/s}$$

Average Speed = Path Length / Time Interval

$$= 360/18 = 20 \text{ m/s}$$

Thus, in this case the average speed is equal to the magnitude of the average velocity.

(b) Average Velocity = Displacement / Time Interval = +240/ (18+6) = +10 m/s

Average Speed = Total Distance / Time Interval = (OP + PQ) / Δt

$$= (360+120)/24s= 20 \text{ m/s}$$

- **INSTANTANEOUS VELOCITY:** The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small. In other words ,

$$\lim_{\Delta t \rightarrow 0} \left(\frac{\Delta x}{\Delta t} \right) = \frac{dx}{dt} = \vec{v}$$

lim stands for the operation of taking limit as $\Delta t=0$ of the quantity on $\Delta t \rightarrow 0$

its right & $\frac{dx}{dt}$ represents the rate of change of position with respect to time, at that instant.

- **Velocity** at any instant is **equal** to the **slope of tangent of displacement time graph**.
- It is the **average velocity** for **infinitely small time interval**.
- Thus instantaneous velocity = $\tan \theta = \frac{dx}{dt}$
- **Magnitude of velocity in rectilinear motion at any time gives us speed of the particle.**
- The **SI unit of Velocity is m/s**.

Example:

The position of an object moving along x-axis is given by $x = a + bt^2$ where $a = 8.5 \text{ m}$, $b = 2.5 \text{ m s}^{-2}$ and t is measured in seconds. What is its velocity at $t = 0 \text{ s}$ and $t = 2.0 \text{ s}$. What is the average velocity between $t = 2.0 \text{ s}$ and $t = 4.0 \text{ s}$?

Ans: The instantaneous velocity is given by $V = \frac{dx}{dt} = \frac{d(a+bt^2)}{dt} = 2bt \text{ m/s}$

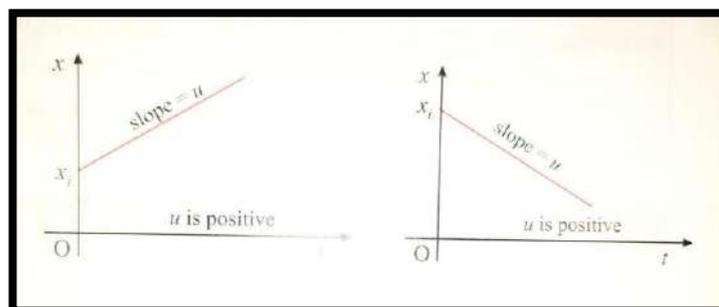
At $t = 0 \text{ s}$, $v = 0 \text{ m s}^{-1}$ and at $t = 2.0 \text{ s}$, $v = 10 \text{ m s}^{-1}$,

$$\text{Average velocity} = \frac{x(4.0) - x(2.0)}{4 - 2} = \frac{a + 16b - a - 4b}{2} = 6 \times 2.5 = 15 \text{ m/s}$$

*Note that for **uniform motion**, velocity is the same as the average velocity at all instants. Instantaneous speed or simply speed is the magnitude of velocity.

Average speed over a finite interval of time is **greater or equal** to the **magnitude of the average velocity**, **instantaneous speed** at an instant is **equal** to the **magnitude of the instantaneous velocity** at that instant.

Position time graph:



AVERAGE AND INSTANTANEOUS ACCELERATION

- **Average Acceleration:** *The rate of change of velocity for a moving particle with respect to time is known as average acceleration.*

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_{final} - v_{initial}}{\Delta t}$$

- **Instantaneous Acceleration:** *The rate of change of velocity for a moving particle at any particular instant is known as instantaneous acceleration. Mathematically its expressed as ,*

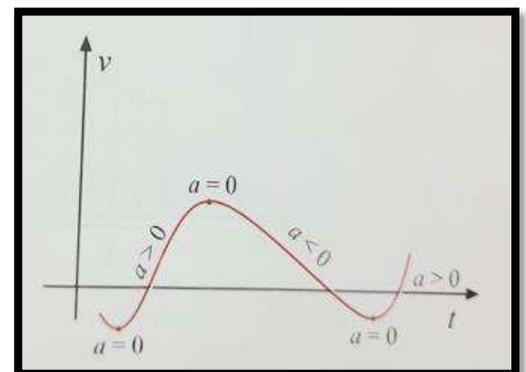
$$\lim_{\Delta t \rightarrow 0} \left(\frac{\Delta v}{\Delta t} \right) = \frac{dv}{dt} = \rightarrow a$$

- It is the **average acceleration** for infinitely small time interval.
- **Acceleration** at any instant is **equal** to the **slope of tangent** of **velocity time graph**.
- Thus instantaneous acceleration = $\tan \theta = \frac{dv}{dt}$
- The SI unit of acceleration is m/s^2 .

$$\text{If } v = f(x) \text{ then } a = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$$

*When displacement is given and we need to calculate a, then we use

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$



Velocity – Time graph

KINEMATICAL EQUATION FOR UNIFORMLY ACCELERATED MOTION

For uniformly accelerated motion, we can derive some simple equations that relate **displacement (x)**, **time taken (t)**, **initial velocity (v₀)**, **final velocity (v)** and **acceleration (a)**.

Relation between final and initial velocities v and v₀ of an object moving with uniform acceleration a can be derived from the relation: $\frac{dv}{dt} = a$

Integrating both sides we get:

$$v = v_0 + at$$

**This is called first equation of motion.*

- The area under the curve represents the displacement over a given time interval.

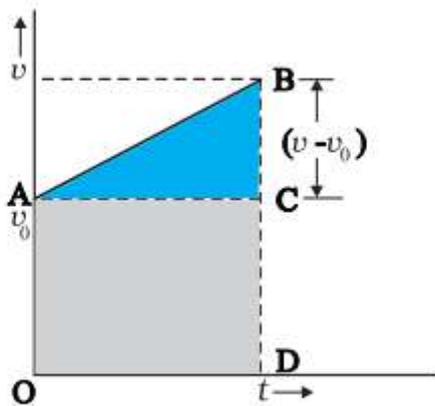


Fig. 3.12 Area under v-t curve for an object with uniform acceleration.

This **2nd equation of motion** is graphically represented in Fig. 3.12.

The **area** under this curve is: Area between instants 0 and t = Area of **triangle ABC + Area of rectangle OACD**

$$X = \frac{1}{2}(v-v_0)t + v_0t$$

$$\text{But } v-v_0 = at$$

Thus,

$$x = v_0t + \frac{1}{2}at^2$$

For constant acceleration only

$$v_{avg} = s/t = (v_0t + \frac{1}{2}at^2)/t$$

$$v_{avg} = u + \frac{1}{2}at = \frac{1}{2}(2u+at) = \frac{1}{2}(u + u + at)$$

$$v_{avg} = \frac{1}{2}(u+v)$$

Where u = initial velocity

And v = final velocity

$$X = vt = \frac{1}{2}(v + v_0)(v - v_0) = \frac{v^2 - v_0^2}{2a}$$

$$v^2 - v_0^2 = 2ax$$

This is **3rd equation** of motion.

All the above sets of equations are obtained by assuming that at $t = 0$ the position of the particle is zero. In case there is an x_0 displacement then the equations of motions are given by:

$$v = v_0 + at$$

$$x = x_0 + v_0t + \frac{1}{2} * (a) * (t^2)$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

Obtaining equations of motion using calculus

$$a = \frac{dv}{dt}$$

$$dv = a dt$$

Integrating both sides

$$\int_{v_0}^v dv = \int_0^t a dt$$

$$= a \int_0^t dt \quad (a \text{ is constant})$$

$$v - v_0 = at$$

$$v = v_0 + at$$

Further,

$$v = \frac{dx}{dt}$$

$$dx = v dt$$

Integrating both sides

$$\int_{x_0}^x dx = \int_0^t v dt$$

$$= \int_0^t (v_0 + at) dt$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

We can write

$$a = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$$

$$\text{or, } v dv = a dx$$

Integrating both sides,

$$\int_{v_0}^v v dv = \int_{x_0}^x a dx$$

$$\frac{v^2 - v_0^2}{2} = a(x - x_0)$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

**The advantage of this method is that it can be used for motion with non-uniform acceleration also.*

FREELY FALLING BODIES

When a body is dropped from some height which is much less than the radius of the earth, it falls freely under gravity with **constant acceleration $g = 9.8 \text{ m/s}^2$** provided the air resistance is negligible.

The same set of kinematical equations are replaced with **$a = g$** and direction of y axis is chosen conveniently.

- **Case-1:** When **y axis is chosen positive** for **vertically downward motion** then we take "**g**" as **positive** since the direction of g is always downwards towards earth.

The equations of motions are:

$$\begin{aligned}v &= v_0 + gt \\h &= v_0t + \frac{1}{2} * (g) * (t^2) \\v^2 - v_0^2 &= 2g(h)\end{aligned}$$

-ve y axis taken as +ve



where h is the vertical displacement

- **Case – 2:** When **+ve y axis** is taken **+ve** for **vertically upward motion** then the equations of motions are :

$$\begin{aligned}v &= v_0 - gt \\h &= v_0t - \frac{1}{2} * (g) * (t^2) \\v^2 - v_0^2 &= -2g (h)\end{aligned}$$

+ve y axis taken as +ve



Where h is the vertical displacement

***Note: Choosing of sign conventions is necessary.** All the **vector quantities** should be assigned the **same convention** and the **convention** should be kept **fixed** throughout the **problem**. For example for case – 1 the displacement, velocity and g has a direction downward n we took downward as positive and rewrote the equations.

Examples :

1. The motion of a particle described by is given by $v = at$, where a is a constant and $a = 9 \text{ m/s}^2$. Find the distance travelled by the particle in first 4 seconds.

Ans : Because for the motion $v = at$, so the acceleration is uniform and is equal to a .

Thus, Distance travelled = $\frac{1}{2}(at^2) = \frac{1}{2}(9 \times 4 \times 4) = 72 \text{ m}$

2. A balloon starting from the ground has been ascending vertically at a uniform velocity for 3 sec. and a stone falls from it reaches the ground in 6 sec. Find the velocity and its height when the stone was let fall. ($g = 10 \text{ m/s}^2$)

Ans : The phrase let fall means the stone is released from rest without any push.

Let u = the constant velocity with which the balloon is going up

Initial velocity of stone (relative to ground) = $0 + u = u$

H = height of the balloon where the stone is dropped

$$H = 3u$$

For the stone : **+ve y axis is +ve** ($s = -ve$ since its falling, $u = +ve$, $g = -ve$)

$$-s = ut - \frac{1}{2}(gt^2)$$

$$-3u = 6u - \frac{1}{2}(10 \times 6 \times 6)$$

$$u = 20 \text{ m/s}$$

$$\text{Height} = 3 \times 20 = 60 \text{ m}$$

3. A ball is thrown vertically upwards with a velocity of 20 m s^{-1} from the top of a multi-storey building. The height of the point from where the ball is thrown is 25.0 m from the ground. (a) How high will the ball rise? and (b) how long will it be before the ball hits the ground? Take $g = 10 \text{ m/s}^2$.

Ans : (a) Let us take the +y-axis in the vertically upward direction with zero at the ground, as shown in Fig. 3.13.

<https://tapoj.com>

$$u = 20 \text{ m/s}$$

$$a = -g = -10 \text{ m/s}^2$$

$$v = 0 \text{ m/s}$$

considering the ball rises to a height y from the point of launch, then using equations of motion

$$v^2 - v_0^2 = -2g(y - y_0)$$

$$0 = 20^2 + 2(-10)(y - y_0)$$

$$(y - y_0) = 20 \text{ m}$$

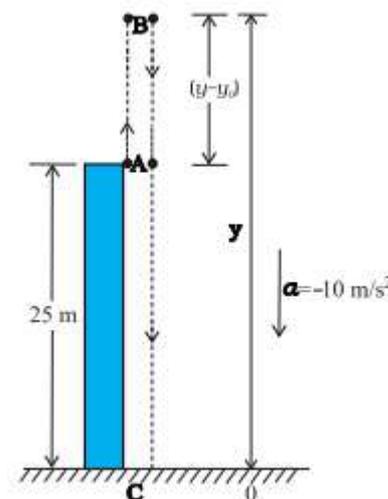


Fig. 3.13

b. the upward motion (A to B) and the downward motion (B to C) and calculate the corresponding time taken t_1 and t_2 . Since the velocity at B is zero, we have:

$$v = u + at$$

$$0 = 20 - 10t_1$$

$$t_1 = 2 \text{ s}$$

This is the time in going from A to B. From B, or the point of the maximum height, the ball falls freely under the acceleration due to gravity. The ball is moving in negative y direction

$$-(y - y_0) = -v_0t - \frac{1}{2}(g)(t^2)$$

$$0 = 45 + \frac{1}{2}(-10)t_2^2$$

$$t_2 = 3 \text{ s}$$

$$\text{Total Time} = (3 + 2) \text{ s} = 5 \text{ s.}$$

Calculating the Displacement during n^{th} second

Displacement of n second - Displacement of $(n-1)$ s

$$= un + \frac{1}{2}(a)(n^2) - \{ u(n-1) + \frac{1}{2}(a)(n-1)^2 \}$$

$$= u(n - n + 1) + \frac{1}{2}(a)\{n^2 - (n-1)^2\}$$

$$s_n = u + \frac{1}{2}(a)(2n-1)$$

Calculation of stopping distance:

When brakes are applied to a moving vehicle, the distance it travels before stopping is called **stopping distance**. It is an important factor for road safety and depends on the initial velocity (v_0) and the braking capacity, or deceleration, $-a$ is caused by the braking.

Let the distance travelled by the vehicle before it stops be d_s . Then, using equation of motion $v^2 = v_0^2 + 2ax$, and noting that $v = 0$, we have the stopping distance

$$d_s = -v^2 / 2a$$

Example:

1. A bullet is fired into a fixed target loses half of its velocity after penetrating 3cm. How much further will it penetrate before coming to rest assuming that it faces constant resistance to motion?

Ans : Let initial velocity be u

After penetrating 3 cm its velocity becomes $= u/2$

From 3rd equation of motion ,

$$v^2 - u^2 = -2a(x)$$

$$(u/2)^2 = u^2 - 2as$$

$$a = u^2/8$$

Let it further penetrate some distance x and stops at some point.

$$0 = (u/2)^2 - 2(u^2/8)x$$

Thus $x = 1\text{cm}$.

2. A body moving with a constant retardation in a straight line travels 5.7m and 3.9m in 6th second and 9th second, respectively. When the body will momentarily come to rest?

Ans : A body is moving with initial velocity u and a n acceleration a and traverses a distance s_n in n^{th} second of its motion.

$$S_n = u + \frac{1}{2}(2n-1)(a)$$

$$5.7 = u + \frac{1}{2}(2 \times 6 - 1)(a)$$

$$3.9 = u + \frac{1}{2}(2 \times 9 - 1)(a)$$

Solving both equations we get ,

$$u = 9 \text{ m/s and } a = -.6 \text{ m/s}^2$$

$$\text{Thus, } 0 = 9 + (-.6) t, \mathbf{t = 9/.6 = 15s}$$

GRAPHICAL REPRESENTATION (MOTION IN A STRAIGHT LINE)

The theory of graphs can be generalised and summarised in following six points. :

- A **linear equation** represents **straight line** e.g. $y = 4x$. $y = kx$ represents a straight line passing through origin in x- y graph.
- $x = k/y$ represents **a rectangular hyperbola** in x-y graph.
- A **quadratic equation** represents a **parabola** in x- y graph .
- **If $z = dy/dx$** , then value of z can be obtained by the **slope of the graph** at that point.
- If $z = xy$, then the value of z between x_1 and x_2 between y_1 and y_2 can be obtained by the **area of graph** between x_1 and x_2 and y_1 and y_2 .

Now we can plot s-t and v-t graphs of some standard results in tabular form.

The graphs given below is drawn for motion in one dimensional with uniform velocity or with constant acceleration.

Tabular form of Graphs:

S. No.	Different Cases	v-t Graph	s-t Graph	Important Points
1.	Uniform motion	 $v = \text{constant}$	 $s = vt$	(i) Slope of v-t graph = $v = \text{constant}$ (ii) In s-t graph $s = 0$ at $t = 0$
2.	Uniformly accelerated motion with $u = 0$ and $s = 0$ at $t = 0$	 $v = at$	 $s = \frac{1}{2}at^2$	(i) $u = 0$, i.e., $v = 0$ at $t = 0$ (ii) a or slope of v-t graph is constant (iii) $u = 0$, i.e., slope of s-t graph at $t = 0$, should be zero
3.	Uniformly accelerated motion with $u \neq 0$ but $s = 0$ at $t = 0$	 $v = u + at$	 $s = ut + \frac{1}{2}at^2$	(i) $u \neq 0$, i.e., v or slope of v-t graph at $t = 0$ is not zero (ii) s or slope of s-t graph gradually goes on increasing
4.	Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at $t = 0$	 $v = u + at$	 $s = s_0 + ut + \frac{1}{2}at^2$	(i) $v = u$ at $t = 0$ (ii) $s = s_0$ at $t = 0$
5.	Uniformly retarded motion till velocity becomes zero	 $v = u - at$	 t_0	(i) Slope of s-t graph at $t = 0$ gives u (ii) Slope of s-t graph at $t = t_0$ becomes zero (iii) In this case u can't be zero
6.	Uniformly retarded then accelerated in opposite direction	 t_0	 t_0	(i) At time $t = t_0$, $v = 0$ or slope of s-t graph is zero (ii) In s-t graph slope or velocity first decreases then increases with opposite sign.

Important points

- Slopes of v-t or s-t graph can never be infinite at any point, because infinite slope of v-t graph means infinite acceleration. Similarly, infinite slope of s-t graph means infinite velocity. Hence these graphs are not possible.

- *At one time only two values of velocity or displacement are not possible.*
- *Different values of displacements in s-t graph corresponding to given v-t graph can be calculated just by calculating the area under v-t graphs.*

Example:

A car accelerates from rest at a constant rate α for some time, after which it accelerates at a constant rate β , to come to rest. If the total time elapsed is t sec evaluate (a) the maximum velocity reached. (b) The total distance travelled.

Ans : (a) Let a car accelerates for time t_1 and decelerates for time t_2 ,

Then , $t = t_1 + t_2 \dots (1)$

$\alpha = \text{Slope of the line OA} = v_{\text{max}}/t_1$

$t_1 = v_{\text{max}}/ \alpha \dots (2)$

$\beta = \text{-ve Slope of line AB} = v_{\text{max}}/t_2$

$t_2 = v_{\text{max}}/ \beta \dots (3)$

From equations (1), (2), (3) we get

$(v_{\text{max}}/ \alpha) + v_{\text{max}}/ \beta$

$v_{\text{max}} = \{ (\alpha \beta) * t \} / (\alpha + \beta)$

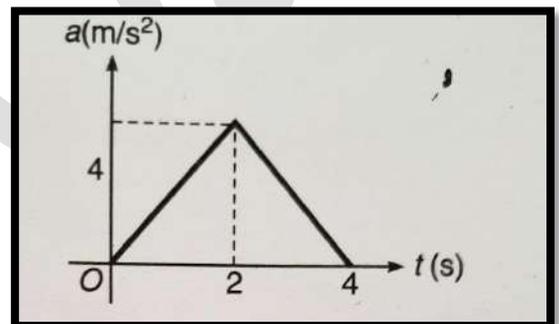
(b) Total distance = Displacement

= Area under v-t graph

= $\frac{1}{2}(txv)$

= $\frac{1}{2}[tx \{ (\alpha \beta) * t \} / (\alpha + \beta)]$

Distance = $\frac{1}{2} * \{ (\alpha \beta) * t^2 \} / (\alpha + \beta)$



MOTION WITH NON-UNIFORM ACCELERATION

- **Acceleration depends on time t**

Steps to find v (t) from a (t)

By definition we have $\frac{dv}{dt} = \vec{a}$

Now integrating both sides,

$$\int_{v(0)}^{v(f)} dv = \int_0^t a(t) dt$$

Where v_0 = Initial velocity at time $t=0$

- **Steps to find x(t) from v(t)**

To get x (t), we put $v(t) = dx/dt$

$dx = v(t)dt$

Integrating both sides,

$$\int_{x(0)}^{x(f)} dx = \int_0^t v(t) dt$$

Where $x(0)$ = Position at $t=0$

Example:

1. A particle is travelling along X axis with an acceleration which varies as $a=-4x$. Derive an expression for $v(x)$. Assume the particle starts from rest $x=1m$. Hence find the maximum possible speed of the particle.

Ans : $a(x) = v \cdot dv/dx$ $v \cdot dv/dx = -4x$

$$\int_0^{v(f)} v dv = -4 \int_1^x x dx$$

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$$v^2/2 = -4(x^2/2 - 1/2)$$

$$v^2 = 4(1-x^2)$$

$$v(x) = \pm 2\sqrt{1-x^2}$$

Speed is maximum for $x=0$

Maximum Speed = 2m/s.

Relative Velocity

The word 'relative' means in relation or in proportion to something else.

Relative motion is the motion as observed from or referred to some system constituting a frame of reference.

The relative velocity of A with respect to B (written as \bar{u}_{AB}) is

$$\bar{u}_{AB} = \bar{u}_A - \bar{u}_B$$

Similarly, relative acceleration of A with respect to B is

$$\bar{a}_{AB} = \bar{a}_A - \bar{a}_B$$

Example:

Two parallel rail tracks run north-south. Train A moves north with a speed of 54 km h^{-1} , and train B moves south with a speed of 90 km h^{-1} . What is the (a) velocity of B with respect to A ?, (b) velocity of ground with respect to B ?, and (c) velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 km h^{-1} with respect to the train A) as observed by a man standing on the ground ?

Ans : Choose the positive direction of x-axis to be from south to north. Then,

$$v_A = + 54 \text{ km/hr} = 15\text{m/}$$

$$v_B = -90 \text{ km/hr} = - 25\text{m/s}$$

Relative velocity of B with respect to A = $v_B - v_A = 40 \text{ m/s}$.

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The train B appears to A to move with a speed of 40 m s^{-1} from north to south.

Relative velocity of ground with respect to B = 2 m/s

In (c), let the velocity of the monkey with respect to ground be v_M . Relative velocity of the monkey with respect to A,

$$v_{MA} = v_M - v_A = -18 \text{ km/h} = -5 \text{ m/s}$$

$$v_M = (15 - 5) \text{ m/s} = 10 \text{ m/s}$$

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